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**ARMATURE
RIPPLE
THEORY**

PHASE 1

Armature Ripple Theory

Phase 1

Scalarization Threshold on a Schwarzschild Background

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Armature Ripple Theory — Phase 1 Scalarization Threshold on a Schwarzschild Background

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Abstract

We present the first computational result for Armature Ripple Theory (ART), a minimal scalar–Gauss–Bonnet extension of General Relativity. Working in the decoupling limit on a fixed Schwarzschild background, we analyze the onset of curvature-induced scalarization by solving the linearized scalar field equation with quadratic coupling to the Gauss–Bonnet invariant.

The resulting boundary-value problem is treated as an eigenvalue problem for a dimensionless coupling parameter. Imposing regularity at the horizon and decay at infinity, we identify a critical coupling

$$\eta_{\text{crit}} \approx 0.18139$$

at which a nontrivial, asymptotically decaying scalar configuration appears.

This solution represents the threshold mode signaling the onset of bifurcation away from the Schwarzschild branch. The result establishes the existence of curvature-triggered scalarization within ART and provides a concrete computational result for subsequent nonlinear analysis.

1. Introduction

Armature Ripple Theory (ART) is a minimal scalar–Gauss–Bonnet (EsGB) extension of General Relativity designed to isolate curvature-induced effects in strong gravity.

In this class of theories, the scalar field couples directly to the Gauss–Bonnet invariant, allowing spacetime curvature to act as an effective source term. In sufficiently strong curvature regimes, this coupling can induce a tachyonic instability, leading to spontaneous scalarization.

This evasion of standard no-hair theorems is enabled by the non-minimal coupling of the scalar field to the Gauss–Bonnet invariant.

The purpose of Phase 1 is to determine whether such an instability arises within the ART framework, and if so, to identify the threshold at which it occurs.

To isolate this mechanism, we work in the decoupling limit, treating the scalar field on a fixed Schwarzschild background. The problem reduces to a linear differential equation with curvature-dependent effective mass, subject to physically motivated boundary conditions.

2. Theoretical Setup

We consider the scalar sector relevant to the decoupling-limit analysis:

$$S = \int d^4x \sqrt{-g} \left[-(1/2)(\nabla\phi)^2 + (\alpha/2) \phi^2 G \right]$$

where:

- ϕ is a real scalar field
- G is the Gauss–Bonnet invariant
- α is a coupling parameter with dimensions of length^2

We take $\alpha > 0$, ensuring that the effective mass squared

$$m^2_{\text{eff}} = -\alpha G$$

is negative in regions of strong curvature, enabling tachyonic instability.

The Gauss–Bonnet invariant is defined as:

$$G = R^2 - 4 R_{\{\mu\nu\}} R^{\{\mu\nu\}} + R_{\{\mu\nu\rho\sigma\}} R^{\{\mu\nu\rho\sigma\}}$$

Variation with respect to ϕ yields the scalar field equation:

$$\square\phi + \alpha \phi G = 0$$

where $\square$ denotes the covariant d'Alembertian operator.

3. Schwarzschild Background

We consider a fixed Schwarzschild spacetime:

$$ds^2 = -(1 - 2M/r) dt^2 + dr^2/(1 - 2M/r) + r^2 d\Omega^2$$

The Gauss–Bonnet invariant for this geometry is:

$$G = 48 M^2 / r^6$$

Defining the horizon radius $r_h = 2M$, this becomes:

$$G = 12 r_h^2 / r^6$$

Substituting into the scalar equation yields a linear differential equation for $\phi(r)$.

4. Dimensionless Formulation

We introduce dimensionless variables:

$$x = r / r_h$$

$$\eta = \alpha / r_h^2$$

The scalar equation reduces to a second-order ordinary differential equation:

$$\phi''(x) + A(x) \phi'(x) + \eta B(x) \phi(x) = 0$$

where $A(x)$ and $B(x)$ are known functions determined by substitution into the scalar field equation; their explicit forms are omitted here for brevity.

The problem is now parameterized entirely by the dimensionless coupling η .

5. Boundary Conditions

Physical solutions must satisfy:

Horizon regularity ($x = 1$)

The scalar field remains finite at the horizon.

Asymptotic decay ($x \rightarrow \infty$)

The scalar field must decay as:

$$\phi(x) \sim C / x$$

The absence of a constant mode is required. This selects the asymptotically decaying branch and excludes the non-decaying mode incompatible with a localized scalar configuration.

6. Eigenvalue Problem

The system defines a boundary-value problem that can be recast as an eigenvalue problem for η .

For generic values of η :

- solutions either diverge or approach a constant at infinity

Only for specific values of η does a regular, decaying solution exist.

We solve this using a standard numerical shooting method, integrating outward from the horizon and tuning η to eliminate the non-decaying asymptotic mode. Convergence is determined by the decay behavior at large radius.

7. Numerical Result

We identify a critical coupling:

$$\eta_{\text{crit}} \approx 0.18139$$

At this value:

- the scalar field is finite at the horizon
- smooth for all $x \geq 1$
- decays asymptotically as $1/x$

This solution represents the threshold mode.

The quoted value is numerical and should be understood as approximate within the resolution of the shooting procedure. The value is specific to the normalization and coupling structure adopted in ART.

8. Interpretation

The existence of a normalizable solution at η_{crit} indicates the onset of a tachyonic instability in the scalar sector.

Physically, this corresponds to a bifurcation point:

- $\eta < \eta_{\text{crit}} \rightarrow$ the Schwarzschild branch is linearly stable against this scalar mode
- $\eta = \eta_{\text{crit}} \rightarrow$ zero mode appears
- $\eta > \eta_{\text{crit}} \rightarrow$ a scalarized branch is expected to emerge

This establishes that ART admits curvature-induced scalarization in the near-horizon strong-curvature regime.

9. Limitations

This analysis is restricted to:

- fixed Schwarzschild background
- linearized scalar field
- no backreaction

The existence and structure of the scalarized branch require solving the fully coupled system.

While this analysis assumes zero angular momentum ($a = 0$), it provides the necessary spherical limit for future extensions to rotating (Kerr) black hole backgrounds.

10. Conclusion

We have demonstrated the existence of a scalarization threshold in Armature Ripple Theory by solving the linearized scalar equation on a Schwarzschild background.

The critical coupling

$$\eta_{\text{crit}} \approx 0.18139$$

defines the onset of instability and provides the foundation for nonlinear analysis.

This result establishes the first concrete computational result within ART.

Bridge to Phase 2

The scalarization threshold identified here motivates the nonlinear construction of scalarized black hole solutions considered in Phase 2.

Appendix: Transmission

First edition published on the Internet Archive, 2026.
Part of the Sum Bunny Canon (Ark Hive Press).

This work was developed with the assistance of computational and AI-based tools. All arguments, structures, and final judgments are the author's.

Hey Josiane,

Sum Bunny Loves You!!!

$$\eta_{(\text{crit})} \approx 0.18139$$

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